WIDEBAND CONDUCTOR LOSS CALCULATION OF PLANAR QUASI-TEM TRANSMISSION LINES WITH THIN CONDUCTORS USING A PHENOMENOLOGICAL LOSS EQUIVALENCE METHOD

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Abstract

Conductor loss calculation based on the incremental inductance rule is not valid if conductor thickness becomes very thin and on the order of the skin depth, such as in monolithic microwave integrated circuits. In this paper, conductor loss of a planar quasi-TEM transmission line with thin conductors is calculated over a broad frequency range using an approach to be called a phenomenological loss equivalence method. The calculated conductor losses of microstrip lines agree well with those calculated using the finite element method. Because of its simplicity, this method should be very useful for computer-aided design of monolithic microwave circuits. In addition, this method can also be applied to very thin and narrow superconductive lines using the complex conductivity based on the two-fluid model.

Introduction

High speed and high integration in modern integrated circuits, especially in monolithic microwave integrated circuits, demand very narrow and thin metal interconnection lines. High-speed electro-optic devices, such as traveling-wave optical modulators[1], use very narrow and thin electrodes in very wide bandwidth. In such small metal lines, the conductor loss becomes dominant and limits the performance of the circuits and the devices. Therefore, wideband calculation of the conductor loss is needed. DC calculation and the incremental inductance method[2], however, cannot be used directly for the wideband loss calculation. The DC calculation assumes a uniform current distribution inside a conductor. On the other hand, the incremental inductance method is valid only if the conductor thickness is several times of the skin depth. A variational formulation of the skin-effect problem[3] has been used to calculate the conductor loss of a single thin strip which has rectangular cross section without substrate and ground plane. The finite element method is also applied to microstrip-like transmission line structures to calculate the ac resistance and reactance[4]. They, however, are not appropriate for computer-aided design implementation, since these methods require extensive formulations and numerical computations. A simple modification of the skin-effect resistance[5] is valid only for a very thin and wide strip.

In this paper we present a simple and practical method for wideband conductor loss calculation of a planar quasi-TEM transmission line. In this method, a planar quasi-TEM transmission line, having a finite conductor thickness on the order of the skin depth, is approximated by an equivalent strip. The equivalent strip has the same DC and penetration-effect resistances as those of the original transmission line. Therefore, the distriduted resistance of the transmission line at a given frequency can be approximately calculated from the equivalent single strip. For superconductive lines, this method can also be applied using the complex conductivity based on the two-fluid model.

Phenomenological Loss Equivalence Method (PEM)

For a given planar quasi-TEM transmission line such as a microstrip line shown in Fig. 1a, the distributed resistance(R) can be calculated using the incremental inductance method[2] if the skin depth(δ) into the conductor is very shallow compared to the conductor thickness(t), or $t/\delta >> 1$. Specifically,

$$R = R_S\, G \quad (\Omega/m) \tag{1}$$

where R_S is the surface resistivity(Ω/square) of the normal conductor and G is a geometric factor in dimension of m^{-1}. Here, G can be found if R is calculated by applying the incremental inductance rule to the structure. For instance, G=(2/strip width) for very wide microstrip lines[6]. The R_S is the inverse of the product($\sigma\delta$) of the conductivity(σ) and the skin depth(δ). The factor G depends only on the cross-sectional geometry of the transmission line, that is, the quasi-static current distribution on the conductor surface.

Under this condition of shallow field penetration($t/\delta >> 1$), we can imagine an equivalent single conducting strip shown in Fig. 1b, which has the same conductivity and distributed resistance(R) as the original transmission line. The surface current on the equivalent strip is assumed to be distributed uniformly in the horizontal direction. By applying the incremental inductance method to the equivalent strip at the same shallow field penetration as the original transmission line, we can obtain the distributed resistance of the equivalent strip by

$$R = R_S / W_e \quad (\Omega/m) \tag{2}$$

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where W_e is the equivalent width and R_s is the same surface resistivity as the original transmission line. Therefore, by equating (1) and (2), we can express the equivalent width at the shallow penetration as

$$W_e = 1 / G \quad (m) \tag{3}$$

Now, if the frequency decreases and the field penetrates into the conductor, the conductor resistance will also depend on the conductor thickness of the transmission line. However, if the current distribution on the surface of the transmission line can be assumed to be unchanged, e.g., in the case of quasi-TEM transmission line, we can still use the equivalent width(W_e) calculated above and include the field penetration effect into the conductor thickness(t_e) of the equivalent strip.

In planar quasi-TEM structures, screening eddy current exponentially decays from each planar conductor surface. The current inside the equivalent strip also decays exponentially with the same skin depth as the original transmission line. At very high frequencies, the currents flow only on the conductor surfaces of the transmission line and the equivalent strip. As the frequency decreases, the field penetrates into the conductors and the current distributions become more uniform. Finally, at DC, the field penetration is maximum and the current distributions are completely uniform inside the conductors of the transmission line and the equivalent strip. Therefore, the equivalent thickness t_e can be approximately calculated by equating the DC resistance of the equivalent strip to the actual DC resistance of the original transmission line because their conductivities and skin depths are assumed to be the same. Namely,

$$R_{DC} = 1 / (\sigma W_e t_e) \quad (\Omega/m) \tag{4}$$

The actual DC resistance can be calculated simply from the cross-section of the original transmission line. For the microstrip case, $R_{DC}=1/(\sigma Wt)$. Finally, using (3) and (4), the equivalent thickness can be expressed as

$$t_e = G / \sigma R_{DC} \quad (m) \tag{5}$$

Here, t_e depends only on the cross-sectional geometry of the original quasi-TEM line since G and σR_{DC} can be calculated from the cross-sectional geometry.

For the equivalent strip, which has the laterally uniform current distribution, the distributed resistance and the internal reactance due to the field penetration can be calculated using the strip width(W_e) and the surface impedance($Z_s{}^t$) of a flat plane conductor with finite thickness[7]. In the surface impedance calculation, the longitudinal current distribution in the vertical direction of the strip is subject to the boundary conditions. From these conditions, the longitudinal current(I) integrated through the strip thickness can be obtained in terms of the longitudinal electric field(E_0) on the strip surface. Therefore, the surface impedance($Z_s{}^t$) of the planar strip with finite thickness(t_e) can be expressed as

$$Z_s{}^t = E_0 / I = (1+j)\, R_s \coth [(1+j)\, t_e / \delta] \quad (\Omega/\text{square}) \tag{6}$$

Since the equivalent strip has the laterally uniform current distribution over the finite width $W_e(=1/G)$, the distributed internal impedance(Z) due to the field penetration becomes $Z_s{}^tG$ as shown in (1).

Ultimately, for a planar quasi-TEM transmission line with finite conductor thickness, the ditributed resistance(R) and the internal reactance($j\omega L_i$) due to the field penetration can be obtained through the equivalent strip as

$$Z=R+j\omega L_i=Z_s{}^t/W_e=(1+j)R_sG \coth[(1+j)GR_s/R_{DC}] \quad (\Omega/m) \tag{7}$$

The propagation characteristics(i.e. attenuation, effective index, and characteristic impedance) of the original transmission line can be readily calculated from a circuit model of transmission line. In the model, the distributed resistance(R) and the internal inductance(L_i) calculated from (7) will be added in series to the external inductance(L) of the transmission line while the shunt capacitance(C) almost remains constant aginst the field penetration.

The usefulness of this method comes from very simple calculations of G and R_{DC} used in (7). The G and R_{DC} can be calculated using empirical formulas for planar transmission lines and from the cross-sections of the transmission lines, respectively.

Using the complex conductivity of superconductor based on the two-fluid model, this method can also be applied to superconductive quasi-TEM lines, where the penetration depths are almost constant with the frequency and the internal resistances are very small compared to the internal reactances. The effective thickness of the equivalent strip can be calculated at infinite field penetration. Especially, this method will be effective for very thin and narrow superconductive lines, where the line thicknesses are not thick enough to apply the incremental inductance method and the simple modification of surface impedance[8] is not accurate due to the lateral surface current spreading at very deep field penetration.

Calculated Results for Microstrip Lines

For microstrip lines having the geometry of Fig.1 scaled by different factors, the disributed resistances are calculated using the phenomenological loss equivalence method(PEM). In Fig. 2 the calculated resistances are compared with published data calculated using the finite element method(FEM)[6]. The calculated attenuations are also compared with the data in Fig.3. They are in good agreement in a wide frequency range and for a wide range of geometrical dimensions. The slight differences at high frequencies are due to the inaccurate calculations of the factor G. That is because the empirical formulas for microstrip[9] used are not accurate at this high ratio of thickness to width of the microstrip. This deficiency can be easily rectified if we use more accurate empirical formulas. The calculated propagation constants and the characteristic impedances are also in very good agreement with the FEM data.

Conclusion

A phenomenological loss equivalence method is proposed to calculate the conductor losses of planar quasi-TEM transmission lines over a wide frequency range. In this method the conductor losses can be readily calculated from a single strip equivalent to the original transmission lines. The calculated results for microstrip line are in good agreement over a broad bandwidth with those calculated using the finite element method. Due to the simplicity of the method, this method should be very useful for computer-aided design of monolithic microwave circuits. Finally, this method can also be applied to superconductive transmission lines using the two-fluid model of superconductor.

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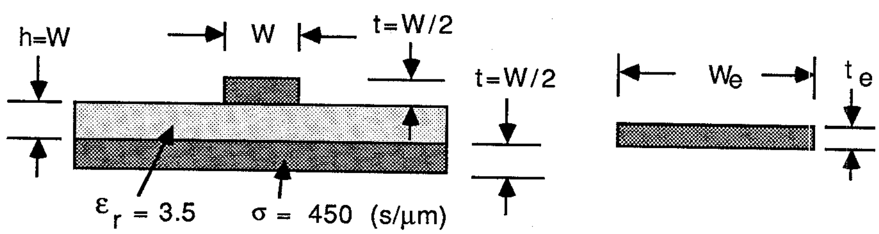

a. Microstrip b. Equivalent Strip

Fig.1 Microstrip line and the equivalent strip

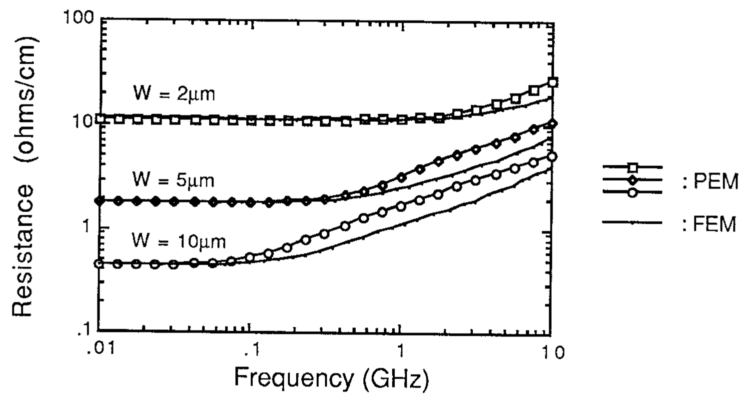

Fig. 2 The distributed resistances of the microstrip line shown in Fig. 1 calculated for different strip widths

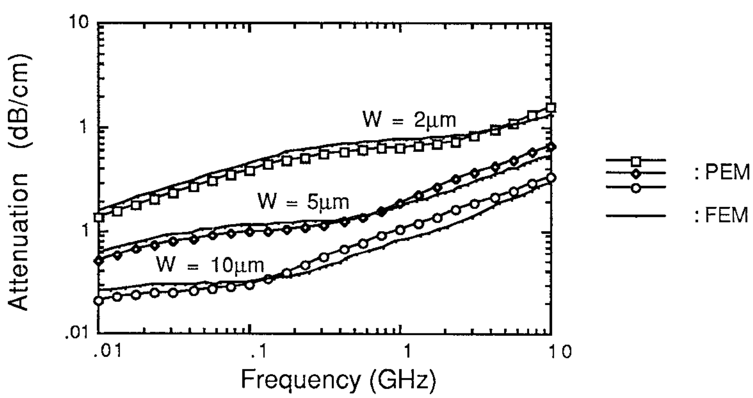

Fig. 3 The attenuations of the microstrip line shown in Fig. 1 calculated for different strip widths